Cambridge Pre-U

Cambridge International Examinations
Cambridge Pre-U Certificate

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FURTHER MATHEMATICS (SHORT COURSE) **1348/01**

Paper 1 Further Pure Mathematics **May/June 2017**

3 hours

Additional Materials: Answer Booklet/Paper
Graph Paper
List of Formulae (MF20)

READ THESE INSTRUCTIONS FIRST

If you have been given an Answer Booklet, follow the instructions on the front cover of the Booklet.
Write your Centre number, candidate number and name on all the work you hand in.
Write in dark blue or black pen.
You may use an HB pencil for any diagrams or graphs.
Do not use staples, paper clips, glue or correction fluid.

Answer **all** the questions.
Give non-exact numerical answers correct to 3 significant figures, or 1 decimal place in the case of angles in degrees, unless a different level of accuracy is specified in the question.
The use of an electronic calculator is expected, where appropriate.
You are reminded of the need for clear presentation in your answers.

At the end of the examination, fasten all your work securely together.
The number of marks is given in brackets [ ] at the end of each question or part question.
The total number of marks for this paper is 120.

The syllabus is approved for use in England, Wales and Northern Ireland as a Cambridge International Level 3 Pre-U Certificate.

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1 Without using a calculator, determine the possible values of a and b for which $(a + \mathrm{i}b)^2 = 21 - 20\mathrm{i}$. [4]

2 The equation $x^3 + 2x^2 + 3x + 7 = 0$ has roots α, β and γ. Evaluate $\alpha^2 + \beta^2 + \gamma^2$ and use your answer to comment on the nature of these roots. [4]

3 **(i)** Sketch the curve with polar equation $r = \dfrac{1}{1 + \theta}$, $0 \leqslant \theta \leqslant 2\pi$. [3]

(ii) Find, in terms of π, the area of the region enclosed by the curve and the part of the initial line between the endpoints of the curve. [3]

4 The curve C has parametric equations $x = \frac{1}{2}t^2 - \ln t$, $y = 2t$, for $1 \leqslant t \leqslant 4$. When C is rotated through 2π radians about the x-axis, a surface of revolution is formed of surface area S. Determine the exact value of S. [7]

5 **(i)** Use the definition $\tanh y = \dfrac{\mathrm{e}^{2y} - 1}{\mathrm{e}^{2y} + 1}$ to show that $\tanh^{-1} x = \frac{1}{2} \ln\left(\dfrac{1 + x}{1 - x}\right)$ for $|x| < 1$. [3]

(ii) Solve the equation $\tanh x + \coth x = 4$, giving your answer in the form $p \ln m$, where p is a positive rational number and m is a positive integer. [5]

6 The curve S has equation $y = \dfrac{x^2 + 1}{(x + 1)^2}$.

(i) Write down the equations of the asymptotes of S. [2]

(ii) Determine $\dfrac{\mathrm{d}y}{\mathrm{d}x}$ and hence find the coordinates of any turning points of S. [4]

(iii) Sketch S. [3]

7 **(i)** Find the value of the constant k for which $y = kx \sin 2x$ is a particular integral of the differential equation $\dfrac{\mathrm{d}^2y}{\mathrm{d}x^2} + 4y = 8 \cos 2x$. [4]

(ii) Solve $\dfrac{\mathrm{d}^2y}{\mathrm{d}x^2} + 4y = 8 \cos 2x$, given that $y = 1$ and $\dfrac{\mathrm{d}y}{\mathrm{d}x} = 1$ when $x = 0$. [7]

8 The line l has equation $\mathbf{r} = \lambda\mathbf{d}$ and the plane Π_1 has equation $\mathbf{r.n} = 35$, where

$$\mathbf{d} = \begin{pmatrix} 2 \\ -1 \\ 2 \end{pmatrix} \quad \text{and} \quad \mathbf{n} = \begin{pmatrix} 6 \\ -2 \\ 3 \end{pmatrix}.$$

(i) **(a)** Determine the exact value of $\cos\theta$, where θ is the angle between $\mathbf{d}$ and $\mathbf{n}$. [3]

(b) Determine the position vector of the point of intersection of l and Π_1. [3]

(c) Determine the shortest distance from O to Π_1. [2]

(ii) The plane Π_2 has cartesian equation $12x - 4y + 6z + 21 = 0$. Determine the distance between Π_1 and Π_2. [3]

9 **(i)** Given that $x \geqslant 1$, use the substitution $x = \cosh\theta$ to show that

$$\int \frac{1}{x^2\sqrt{x^2-1}}\,\mathrm{d}x = \frac{\sqrt{x^2-1}}{x} + C$$

where C is an arbitrary constant. [4]

(ii) By differentiating $\sec y = x$ implicitly, show that $\dfrac{\mathrm{d}}{\mathrm{d}x}(\sec^{-1}x) = \dfrac{1}{x\sqrt{x^2-1}}$ for $x \geqslant 1$. [4]

(iii) Use integration by parts to determine $\displaystyle\int \frac{\sec^{-1}x}{x^2}\,\mathrm{d}x$ for $x \geqslant 1$. [4]

10 **(i)** Express $\dfrac{1}{(k-1)k(k+1)}$ in partial fractions. [3]

(ii) Let $S_n = \displaystyle\sum_{k=3}^{n} \frac{1}{(k-1)k(k+1)}$ for $n \geqslant 3$. Use the method of differences to show that

$$S_n = \frac{1}{12} - \frac{1}{2n(n+1)},$$

and write down the limit of S_n as $n \to \infty$. [5]

(iii) Given that k is a positive integer greater than 1, explain why $\dfrac{1}{k^3} < \dfrac{1}{(k-1)k(k+1)}$. [1]

(iv) Show that $\dfrac{27}{24} < \displaystyle\sum_{k=1}^{\infty} \frac{1}{k^3} < \frac{29}{24}$. [3]

11 **(i)** **(a)** Given $\mathbf{A} = \begin{pmatrix} a & b \\ c & d \end{pmatrix}$ and $\mathbf{B} = \begin{pmatrix} e & f \\ g & h \end{pmatrix}$, work out the matrix $\mathbf{AB}$ and write down expressions for $\det \mathbf{A}$ and $\det \mathbf{B}$. [2]

(b) Verify, by direct calculation, that $\det(\mathbf{AB}) = \det \mathbf{A} \times \det \mathbf{B}$. [2]

Let S be the set of all 2×2 matrices with determinant equal to 1.

(ii) Show that $(S, \times_{\mathrm{M}})$ forms a group, G, where $\times_{\mathrm{M}}$ is the operation of matrix multiplication. [You may assume that $\times_{\mathrm{M}}$ is associative.] [5]

(iii) **(a)** Show that $\mathbf{K} = \begin{pmatrix} 1 & \mathrm{i} \\ \mathrm{i} & 0 \end{pmatrix}$ is an element of G. [1]

Let H be the smallest subgroup of G that contains $\mathbf{K}$ and let n be the order of H.

(b) Determine the value of n. [3]

(c) Give a second subgroup of G, also of order n, which is isomorphic to H. [2]

12 For each positive integer n, the function F_n is defined for all real angles θ by

$$\mathrm{F}_n(\theta) = c^{2n} + s^{2n}$$

where $c = \cos\theta$ and $s = \sin\theta$.

(i) Prove the identity

$$\mathrm{F}_{n+2}(\theta) - \tfrac{1}{4}\sin^2 2\theta \times \mathrm{F}_{n+1}(\theta) \equiv \mathrm{F}_{n+3}(\theta). \qquad [4]$$

Let z denote the complex number $c + \mathrm{i}s$.

(ii) Using de Moivre's theorem,

(a) express $z + z^{-1}$ and $z - z^{-1}$ in terms of c and s respectively, [3]

(b) prove the identity $8(c^6 + s^6) \equiv 3\cos 4\theta + 5$ and deduce that

$$c^6 + s^6 \equiv \cos^2 2\theta + \tfrac{1}{4}\sin^2 2\theta. \qquad [7]$$

(iii) Prove by induction that, for all positive integers n,

$$c^{2n+4} + s^{2n+4} \leqslant \cos^2 2\theta + \frac{1}{2^{n+1}}\sin^2 2\theta.$$

[You are **given** that the range of the function F_n is $\dfrac{1}{2^{n-1}} \leqslant \mathrm{F}_n(\theta) \leqslant 1$.] [7]